The assignment is due at the beginning of class on April 26, 2006.

Problem 1 (10 points) 1. Find a function whose domain is the set of real numbers $\mathbb{R}$ and whose range is the set of rational numbers $\mathbb{Q}$.

2. Find a function whose domain is the set of natural numbers $\mathbb{N}$ and whose range is the set of integers $\mathbb{Z}$.

Problem 2 (10 points) Prove or disprove: If $f$ is a function from $A$ to $B$, and if $f^{-1}$ denotes its inverse relation, then

1. $(x, x) \in (f^{-1} \circ f)$ for all $x \in A$.

2. $(x, x) \in (f \circ f^{-1})$ for all $x \in B$.

Problem 3 (10 points) 1. Find functions $f : B \to C$, $g : A \to B$ and $h : A \to B$ such that $f \circ g = f \circ h$, yet $g \neq h$.

2. Suppose $f : A \to B$ is a function with range$(f) = B$. Prove or disprove: If $g : B \to C$ and $h : B \to C$ satisfy $g \circ f = h \circ f$, then $g = h$.

Problem 4 (10 points) Let $A$ and $B$ be sets with $m$ and $n$ elements respectively. What is the probability that a relation from $A$ to $B$ chosen at random is a function?