Abandoning Dead Ends: Embracing Lively Beginnings

By Edward B. Burger and Michael Starbird

Question to typical college graduate majoring in the liberal arts: You graduated from college 15 years ago. What was the final mathematics course you took?

Former student: Pre-calculus.

Interviewer: What was your final literature course?

Former student: Pre-Shakespeare.

Pre-calculus should never be a final mathematics course. Pre-calculus and algebra provide essential techniques for a student who requires those specific skills in a technical profession or in subsequent courses. However, technical facility with manipulating algebraic expressions is soon forgotten by anybody who does not practice such manipulations. Algebra is not a used or useful life skill for most people.

Having students end their mathematical education with college algebra or pre-calculus may be the result of a hierarchical view of mathematics. In the eyes of many, mathematics is an edifice built on a foundation that includes algebra and pre-calculus. Students often absorb this idea: they see mathematics as an unending string of courses that starts with arithmetic and progresses relentlessly through high school algebra, geometry, pre-calculus, calculus, and so on ad infinitum or perhaps ad nauseam. Each subsequent course is viewed as dependent on the previous one, and there is no independent payoff from any particular course. It is just the next step in the weary journey. This curricular paradigm results in students traveling along this road as far as they can or wish to. Wherever they stop, their mathematical education ends and is quickly forgotten. If students stop prematurely (as all liberal arts majors will), then they summarize their mathematical education as, "I got as far as pre-calculus."

Question to typical college graduate majoring in the liberal arts: How would you describe the mathematics component of your college education?

Good answer: Important, mind-opening, surprising, interesting, full of useful techniques of effective thinking, life-changing, culturally significant, educationally central.

Actual answer: I completed my math requirement.

Follow-up question to likely answer: What did you learn?

Follow-up answer: I got a C.

We have a tradition of letting students end their mathematics education without seeing any truly enticing mathematical notions. Why do we persist in throwing away golden opportunities to bring intriguing, deep, and valuable ideas from mathematics to students? Having students struggle up the first two rungs of a 100-rung ladder that they will never climb is a curricular strategy born of habit rather than thought. Let's not have students' mathematical journeys end in dead ends.

Things that are not used are not useful to the non-user. Mathematical methods such as linear programming and matrix manipulation are applied in the real world. However, does it follow that it is important to teach such techniques to people who will not use them, or use them only indirectly? Of course, we should not go out of our way to avoid real applications of mathematics. However, the question for students in non-technical majors is whether mathematics can be of personal value to them. The argument that everyone might find uses for algebraic skills in actual life (to figure how to amortize a loan, for example) is extremely weak. Let's be honest with ourselves. How often in our everyday lives do we need to find the foci of a hyperbola? How often does anyone in a non-technical profession multiply matrices?

Question to typical college graduate majoring in the liberal arts: What is mathematics?

Student: Mathematics is problems.

Interviewer: *Do you like problems?*

Student: No.

Interviewer: *What is mathematics good for?*

Student: Math gives the formulas you use to do problems at the end of each section of the textbook.

Question to the student's mathematics teacher: What do you teach your students?

Teacher: I teach them how to solve problems.

Interviewer: *Where do you get these problems?*

Teacher: At the end of each section of the textbook.

Students have clear expectations as they enter a mathematics course. They expect to have homework consisting of problems from the textbook. They expect those problems to be identical to the problems that are worked out in the section, with only the numbers changed.. They expect tests consisting of problems similar to ones at the ends of sections, with the numbers changed once again. Most teachers follow this model of teaching.

There is nothing wrong with this model if the goal is to have students mimic worked-out exercises from textbook sections. When we teach liberal arts students, we need to think honestly about what we hope students will take with them from the course and then emphasize those lessons in our teaching. We can highlight methods of thought that arise within mathematics and that are valuable to students as they solve problems not in textbook sections, but in their real, complex lives.

Mathematics courses can touch the lives of our students. Let's not have modest goals. We want our students to look at their lives, their habits of thought, and their world in a new and deeper way. That is the goal of the whole of education, and mathematics can play a central role in allowing all students to grow intellectu-

ally. All teachers should share the goal of improving the lives of their students-that is the purpose of education. However, it seems that the mathematics community has adopted a rather limited self-image about how profoundly our courses can affect students.

Mathematics involves penetrating techniques of thought that all people can use to solve problems, analyze situations, and sharpen the way they look at their world. Our mathematics courses should emphasize basic strategies of thought and analysis. Part of the power of mathematics lies in its inexorable quest for elegance, symmetry, order, and grace. Seeking pattern, order, and understanding is a transforming process that mathematics can help students develop. These and other strategies of thinking that have led to great ideas in mathematics can have their greatest value to people in making real-life decisions and facing situations that are completely outside mathematics. It is a crime for any student to leave a mathematics course with the impression that mathematics is a collection of mindless, rote procedures.

Stressing some basic "life lessons," inspired by mathematical thinking, can empower students to grapple with and conquer the problems and issues that they all face in their lives from love to business, from art to politics. If such basic, effective strategies of thought allow students to conquer infinity and the fourth dimension, then what can't they do? By emphasizing the process by which mathematicians create and discover concepts, we can find powerful strategies of thinking that are effective everywhere.

Effective thinking surely is the main goal of any mathematics course. Students must always realize that their primary job is to think and to develop habits of thinking that are illustrated by the mathematics.

Coming to grips with hard ideas is not smooth sailing. Sometimes students will confront issues that start beyond their grasp, but our challenge is to teach students how to make vagueness turn into clarity and confusion evolve into comprehension. The journey to true understanding can be difficult and frustrating, but if we can engineer a happy ending, the challenges of that journey will be one of the highlights of the course.

Mathematics is not to be viewed from afar. Students should personally engage in the thinking of mathematical ideas. When we present the idea of infinity, we must expose the thinking process by which a systematic exploration of a simple everyday concept of equality (that is, one-to-one correspondence) leads to an intellectual triumph. The idea of exploring consequences of clearly stated simple ideas is a lesson that can potentially last longer than the recollection of Cantor's diagonalization process.

Somehow, we must invite our students to answer questions and not be afraid to make mistakes. Failing is the only way to learn. It is much better to guess a wrong answer than not to think about the question at all. But that idea is one that requires acculturation and must be built into the method of instruction. It is not in keeping with most of their mathematical experience. In short, failure must be encouraged.

Top 10 Lessons for Life

1. Just do it.
2. Make mistakes and fail, but never give up.
3. Keep an open mind.
4. Explore the consequences of new ideas.
5. Seek the essential.
6. Understand the issue.
7. Understand simple things deeply.
8. Break a difficult problem into easier ones.
9. Examine issues from several points of view.
10. Look for patterns and similarities.

Mathematics contains some of the greatest ideas of humankind-ideas comparable to the works of Shakespeare, Plato, and Michelangelo. Mathematical ideas help shape history, and they can add texture, beauty, and wonder to the lives of all students. Deep, fascinating concepts in mathematics can be authentically presented to liberal arts students. It is an exciting adventure for students and their instructors to grapple with notions of infinity, the fourth dimension, chaos, fractals, coincidences, and the random.

Mathematics has a great story to tell and we should be forgiven for any excesses of unbridled enthusiasm as we open doors for students to enter into worlds of profound interest and intrigue, but we should not be forgiven for making mathematics appear mundane and boring. This journey of the imagination and the mind should be fun. If students don't enjoy the course, then their thinking will end at the end of the final. Knowledge comes and goes, but hatred lasts forever. Creating a positive attitude for a lifetime has a continued, incrementally valuable effect. It influences not only the students in the class now, but it influences their presentation of a mathematical attitude to their children in the future. If either the teacher of the class or the students don't enjoy the experience, the class has not realized its true potential. Students should leave the class with the impression that there is much more fascination remaining to be discovered and learned. Let's leave them with a thirst for more.

Shall any gazer with mortal eyes
Or any searcher know with mortal mind
Veil after veil will lift but there must be
Veil after veil behind.

-Sir Edwin Arnold

Interviewer: *What was the biggest idea you learned in college?*

Former student: Infinity.

Interviewer: *What was the most mind-expanding concept you learned in college?*

Former student: The fourth dimension.

Interviewer: *What class in college most improved your ability to think?*

Former student: My math class. ■

Edward Burger and Michael Starbird are co-authors of The Heart of Mathematics: An invitation to effective thinking *(textbook with kit). Both have been awarded numerous teaching awards over the years. Most recently, Starbird has won the prestigious 2000 Friar Society Centennial Teaching Fellowship and Burger has been named a winner of the 2001 MAA Deborah and Franklin Tepper Haimo Award.*