Two First Year Seminar Course Designs for Science and Mathematics Majors

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Overview

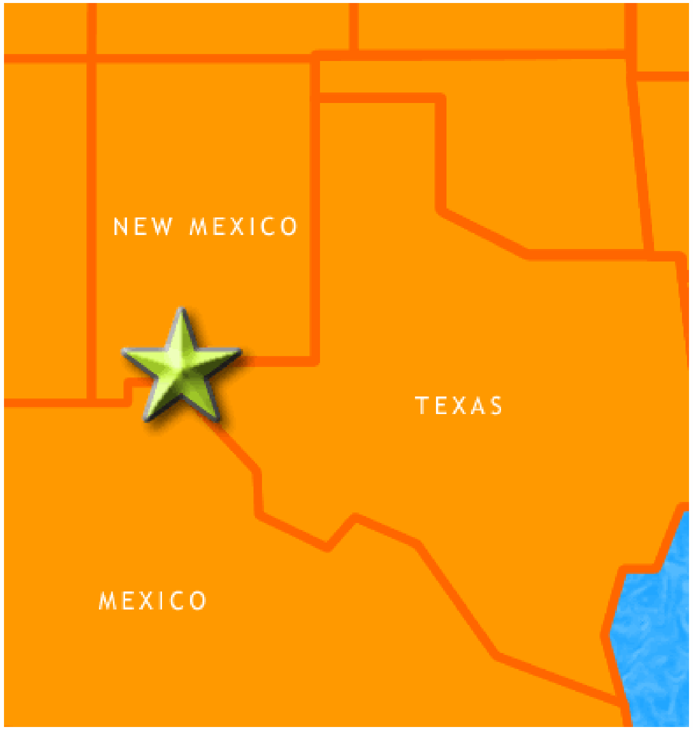
1. The University of Texas at El Paso
2. The FYS at UTEP
3. Design #1: “Scientific Revolutions”
4. Design #2: “Boolean Algebras”

UTEP Student Profile

About 22,000 students

(17,000 UG and 5,000 GR)

- 24 years of age (undergraduate average)
- 76% Hispanic
- 55% female
- 97% from El Paso County commuting daily
- 81% employed
- 55% first generation university students



UNIV 1301

- Started more than 10 years ago
- Required course for all incoming freshmen (there is an alternative course UNIV 2350 for transfer students)
- About 1300 students take the course in the fall semester, 150 in the spring
- 28 students per section

UNIV 1301 – Course Goals

Students will:

- examine the roles and responsibilities crucial for success in college.
- practice essential academic success skills.
- build a network of faculty, staff and peers.
- assess and understand their interests, abilities, and values.
- become involved in UTEP activities and utilize campus resources.

UNIV 1301 – Course Setup

The course consists of two parts:

- About 50% of the course is devoted to student “survival” skills.
- The rest of the course is dedicated to a “theme”, picked by the instructor.

- Some course sections are intended for all majors, some have a restricted audience.
- The instructional team consists of the instructor, a peer leader, a librarian and an academic advisor.

Theme: Scientific Revolutions

- Intended audience: STEM majors simultaneously taking a developmental mathematics course

Theme: Scientific Revolutions

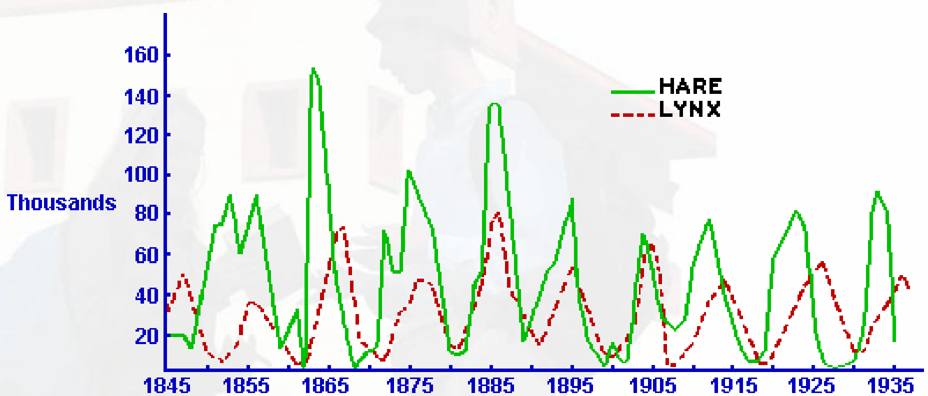
Science history as a "hook" for studying functions via first order difference equations:

"The simplicity of nature is not to be measured by that of our conceptions. Infinitely varied in its effects, nature is simple only in its causes, and its economy consists in producing a great number of phenomena, often very complicated, by means of a small number of general laws."

Theme: Scientific Revolutions

Topics:

- Galileo – linear versus quadratic functions
- Copernicus/Kepler – power functions
- Fibonacci – exponential growth, quadratic functions, limits
- Logistic growth, predator-prey models
- Mendel and Darwin/Hardy-Weinberg – probability, limits



Theme: Scientific Revolutions

Extensive use of Excel spreadsheets:

• They are the perfect tool to visualize the solutions of difference equations

• They allow to cover more advanced subjects by reducing the amount of algebra needed by the students

Example: Geometric Progression

Student Activity

- Fill in the missing data in the table on the right such that the y-data are in geometric progression:

x	y
0.0	1.000
0.5	
1.0	2.000
1.5	
2.0	4.000
2.5	
3.0	8.000

Example: Exponential Growth

Fibonacci Numbers

n	R(n)	R(n)/R(n-1)	n	R(n)	R(n)/R(n-1)
0	1		11	144	1.6179775
1	1	1.0000000	12	233	1.6180556
2	2	2.0000000	13	377	1.6180258
3	3	1.5000000	14	610	1.6180371
4	5	1.6666667	15	987	1.6180328
5	8	1.6000000	16	1597	1.6180344
6	13	1.6250000	17	2584	1.6180338
7	21	1.6153846	18	4181	1.6180341
8	34	1.6190476	19	6765	1.6180340
9	55	1.6176471	20	10946	1.6180340
10	89	1.6181818	21	17711	1.6180340

Example: The Golden Ratio

Cheating, by assuming that the ratio r of consecutive terms is eventually constant, we can compute r:

- r = R(n+2)/R(n+1) = R(n+1)/R(n)
- Using R(n+2)=R(n+1)+R(n), we obtain

  $r = 1 + 1/r$, i.e. $r^2 - r - 1 = 0$

- Solving for r yields the “Golden Ratio” as the positive solution:

$$r = \frac{1}{2}(1+\sqrt{5}) = 1.61803399$$

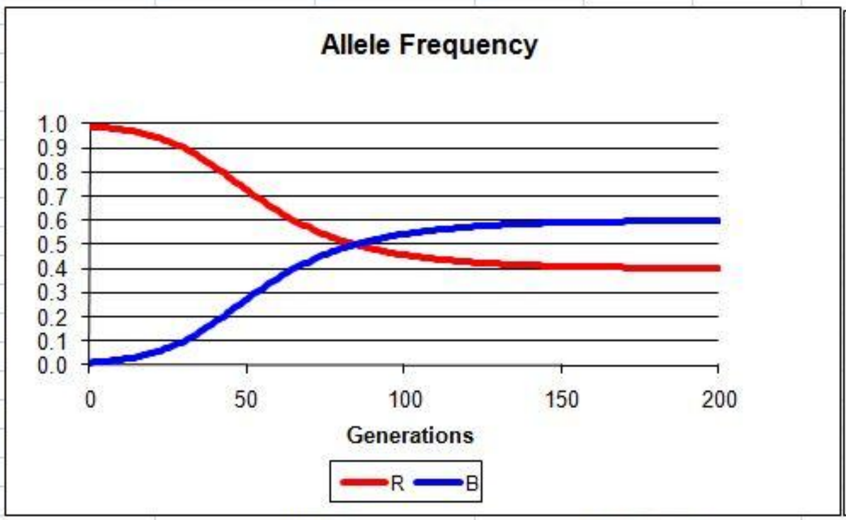
Example: Hardy-Weinberg

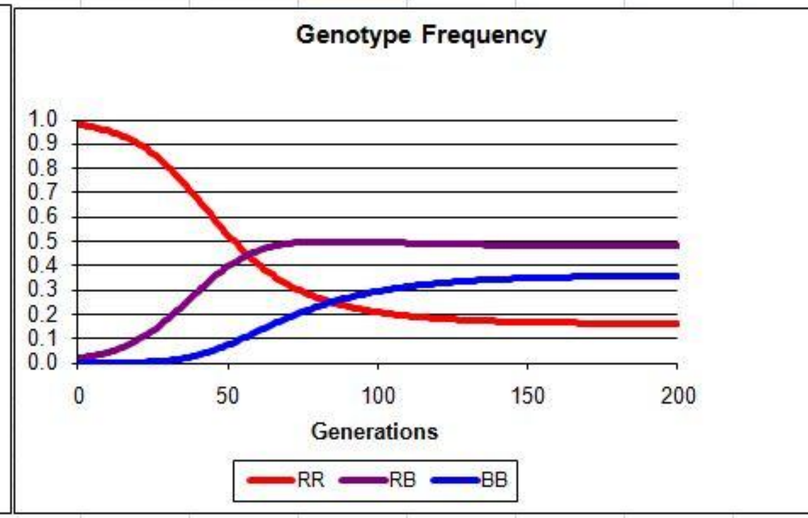
J3 =1-J2

The Hardy-Weinberg Law with Natural Selection

Change the pale blue cells to try different scenarios.

initial allele ratio	R	0.990
	B	0.010
relative survival rates	RR	1.000
	RB	1.090
	BB	1.030





	Alleles		Genotypes			
Generation	R	B	RR	RB	BB	
0	0.990	0.010	0.980	0.020	0.000	1.002
1	0.989	0.011	0.978	0.022	0.000	1.002
2	0.988	0.012	0.976	0.023	0.000	1.002
3	0.987	0.013	0.974	0.025	0.000	1.002
4	0.986	0.014	0.972	0.028	0.000	1.002
5	0.985	0.015	0.970	0.030	0.000	1.003
6	0.984	0.016	0.967	0.032	0.000	1.003

Theme: Boolean Algebras

- Intended audience: Math majors simultaneously taking a Precalculus or Calculus course

- Objective: Expose students early to the method of proof and the study of axiomatic systems

$\wedge$	$y=0$	$y=1$
$x=0$	0	0
$x=1$	0	1

$\vee$	$y=0$	$y=1$
$x=0$	0	1
$x=1$	1	1

$\rightarrow$	$y=0$	$y=1$
$x=0$	1	1
$x=1$	0	1

$\oplus$	$y=0$	$y=1$
$x=0$	0	1
$x=1$	1	0

Figure 1. Truth tables

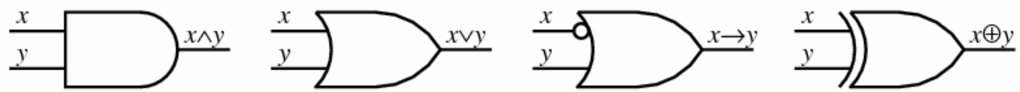


Figure 2. Logic gates

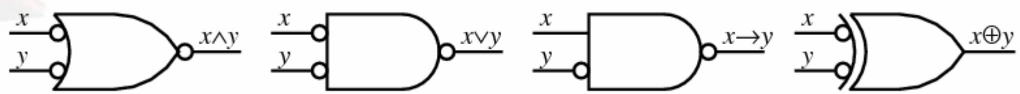
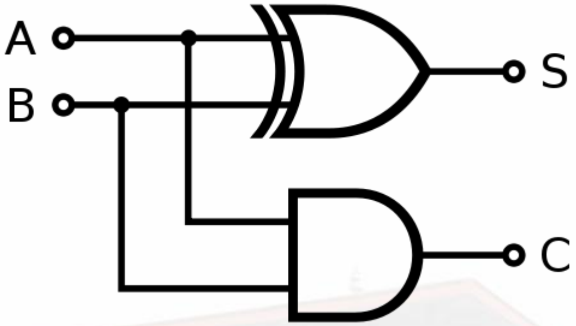
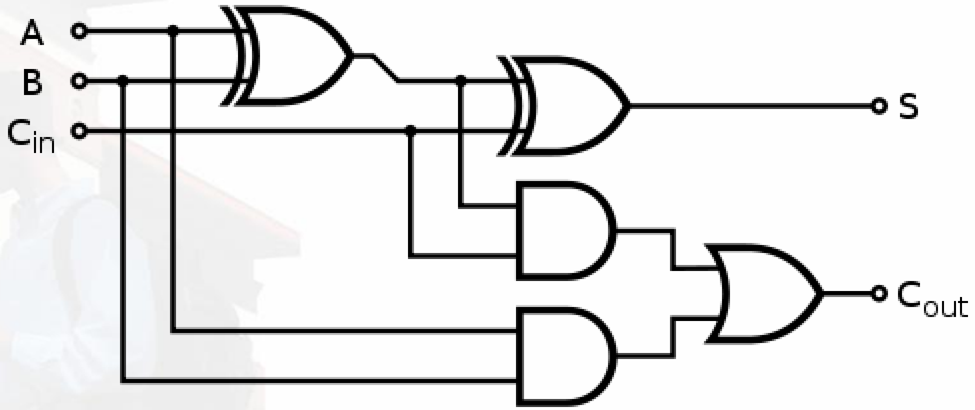


Figure 3. De Morgan equivalents

Figure 4. Venn diagrams

Example: Binary Adders





Example: Atoms

Let $\mathcal{B}$ be a Boolean Algebra with null-element N, partially ordered by $\preceq$. We say that $A \in \mathcal{B}$ is an ATOM of $\mathcal{B}$ if N is the immediate predecessor of A.

1. Find all atoms of $\mathcal{P}(\{1, 2, 3, 4\})$.
2. Find all atoms of $\mathcal{D}_{42}$.
3. Assume additionally that $\mathcal{B}$ has finitely many elements. Show that for every $B \in \mathcal{B}$ with $B \neq N$ there is an atom A such that $A \preceq B$.

Example: Classification

The next problem is the finite version of a general representation theorem for Boolean Algebras, proved by the American mathematician Marshall H. Stone *(1903–1989):*

Let $\mathcal{B}$ be a finite Boolean Algebra with k atoms for some $k \in \mathbb{N}$, and let $\mathcal{A}$ denote the power set of the set of all atoms of $\mathcal{B}$. Given an element B in $\mathcal{B}$, we let

$$\alpha(B) = \{A \in \mathcal{B} \mid A \preceq B \text{ and } A \text{ is an atom of } \mathcal{B}\}.$$

1. Show that the function $\alpha : \mathcal{B} \to \mathcal{A}$ is a bijection.
2. $\mathcal{B}$ has 2^k elements.
3. Show that the identities $\alpha(B \sqcup B') = \alpha(B) \cup \alpha(B')$ and $\alpha(B \sqcap B') = \alpha(B) \cap \alpha(B')$ hold for all $B, B' \in \mathcal{B}$.

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