PRIMUS, 19(6): 510–518, 2009

ISSN: 1051-1970 print / 1935-4053 online
DOI: 10.1080/10511970802245501

One Model of a Capstone Course for Preservice High School Mathematics Teachers

Matthew S. Winsor

Abstract: This article describes the goals of a capstone course for secondary preservice mathematics teachers and how those goals are achieved. Moreover, student reaction to the course will be examined.

Keywords: Preservice teachers, capstone course, content knowledge, mathematical knowledge for teaching, teacher preparation.

1. INTRODUCTION

Reports from the National Council of Teachers of Mathematics (NCTM) and the Mathematical Association of America (MAA) indicate that as a result of changes in the way mathematics is being taught in secondary schools, teachers need a more thorough preparation in mathematics [4, 14, 17, 18]. In response to the need for improving preservice teachers' content knowledge, *The Mathematical Education of Teachers* (MET) [4] publication charges mathematics departments with supporting the development of ''a capstone course sequence for teachers in which conceptual difficulties, fundamental ideas, and techniques of high school mathematics are examined from an advanced standpoint'' [4, p. 39].

In response to the MET call for course development, the author created a mathematics capstone course for preservice high school mathematics teachers attending The University of Texas at El Paso (UTEP). There are roughly 19,000 students at UTEP, with about 100 studying to be secondary mathematics teachers. This program requires a BA in mathematics with a minor in secondary education.

The final mathematics class in the preservice teachers degree program is now the capstone course that was developed. Students take their pedagogy

Address correspondence to Matthew S. Winsor, Department of Mathematics, Illinois State University, Campus Box 4520, Normal, IL 61790-4520, USA. E-mail: mwinsor@ilstu.edu

classes after they have taken the capstone course. In fact, students can only take their education courses after they have finished their content courses and are accepted into the student internship program. The question of mathematical pedagogy does naturally arise in the capstone course as the class focuses on mathematics that is taught in grades 9–12. Because preservice teachers have pedagogy courses in their education minor, the capstone course does not explicitly teach pedagogy nor are students graded on their knowledge of pedagogy, but the instructor does discuss pedagogy when it lends to students mathematical learning.

The text for the course is *Mathematics for High School Teachers: An Advanced Perspective* [21]. The course is designed such that each class has problems that are to be solved and solutions defended by students working in groups. The classroom atmosphere is such that students feel comfortable making their mistakes and learning from them. In fact, the instructor of the course celebrates errors as opportunities for learning [1]. The learning goals of the capstone course at UTEP are three-fold:

1. Students will make connections between their undergraduate mathematics education and the mathematics they will teach as high school mathematics teachers.
2. Students will start to gain a profound understanding of the fundamental mathematics [15] found in the high school curricula.
3. Students will begin to think like a teacher, or in other words, gain mathematical knowledge for teaching [11].

The remainder of this article provides a snapshot of the methods used to achieve the course goals.

2. CONNECTING UNDERGRADUATE MATHEMATICS EDUCATION TO THE TEACHING OF HIGH SCHOOL MATHEMATICS

Preservice teachers often do not see the connections between their undergraduate mathematics courses and their future teaching assignment. One preservice teacher noted that, ‘‘. . . it's so hard to make connections sometimes with a typical college math class and then connecting it back to maybe teaching in high school. And just because you can get so far above your high-school level.’’ [24, p.120]. The following is one example of how connections are made between college level mathematics and high school level mathematics.

Many new high school mathematics teachers are assigned to teach algebra I. One central topic of algebra I is linear functions. Many states in their standards for students expect that students will be able to solve linear equations upon completion of algebra I (e.g., Texas [25]). But many new mathematics teachers do not know the mathematical foundations that allow students to solve linear equations. In the capstone course, students are helped to recognize that indeed they have studied the necessary mathematics to justify the algorithmic process for solving linear equations.

In the capstone course, students are given the equation $a + x = b$ and are asked if $a, b, x \in Z$, does there exist a solution to the equation? Then students are asked if a, b, and x are elements of the odd integers, does there exist a solution to the equation. Most students can see that there exists a solution to the first example, but not the second.

Students are asked to consider why the difference exists in the two situations. Many of the students cannot explain why there is a difference so the instructor gives them another activity. The instructor asks students to solve $a + x = b$ in the most rigorous way they know how. Because students have become adept at solving linear equations, some of the steps are not transparent to them. One step in particular is when you add $-$a to both sides of the equation $a + x = b$. Students do not see that $-a + (a + x) = -a + b$ is the next step in the solution and that they need to use the associative property to combine $-$a and a.

As students work through the ''rigorous'' solution to the linear equation, they list the properties that were used on the board. Those properties are that the associative property holds for the set, each element of the set has an inverse, there is an element in the set that serves as an identity, and that the set is closed for the operation of addition. The natural question then arises, ''Where have you seen these properties together before?'' Students realize that they used these properties in unison when they learned about groups in abstract algebra. Many students note that they had never made this connection between abstract algebra and algebra I while participating in their abstract algebra course.

Another connection between university mathematics and the mathematics taught in high school is found in solving inequalities. Students know that when solving inequalities of the form $-x < a$, where a is an element of the real numbers, that the inequality sign changes direction when multiplying both sides of the inequality by -1. Unfortunately, this is another area where students have accepted the truth of the mathematics without understanding why it works. In the capstone course, students are introduced to the following theorem that gives a rationale for changing the direction of the inequality sign:

For any continuous real functions (*functions whose domain and range are subsets of the real numbers*) f and g, with domain D: If h is strictly decreasing on the intersection of $f(D)$ and $g(D)$, then $f(x) < g(x) \Longleftrightarrow h(f(x)) > h(g(x))$. [21, p.169]

This theorem above requires students to access their prior mathematical knowledge. First, Students must understand the concept of a continuous function. The continuous functions are examined in various mathematics courses in the college curriculum, the first being Calculus 1. Next, students need to understand the concept of strictly decreasing functions. In Calculus 1 students learn methods to determine whether or not a function is strictly increasing/decreasing as well as what it means for a function to be strictly increasing/decreasing. Another necessary piece of prior knowledge is that of the composition of functions. Students at UTEP closely examine the concept of the composition of functions in the upper division mathematics course entitled Principals of Mathematics. Furthermore, students revisit the concept of composition of functions in the capstone course. When examining this theorem, I help students appreciate the necessity to remember and understand mathematics from their previous courses. This is often a shock to my students, who admit that often times they just memorize the material long enough to pass the exam and then forget the mathematics soon thereafter.

3. GAINING A PROFOUND UNDERSTANDING OF FUNDAMENTAL MATHEMATICS IN THE HIGH SCHOOL CURRICULUM

It is essential that teachers have a profound understanding of the mathematics they will teach [2, 3, 15, 16, 19, 22]. Preservice mathematics teachers need the opportunity to develop a deep conceptual learning of mathematics in a collaborative, constructive environment [10]. Moreover, preservice teachers should have the opportunity to relearn mathematics in a manner where they are expected to make conjectures, communicate solution methods, and justify mathematical reasoning [2, 5, 23].

The instructor of the capstone course endeavors to give students the opportunity to examine fundamental mathematics concepts deeply. One such concept is the concept of function. Functions are seen by many as one of the most significant and central concepts in mathematics [6, 9, 12, 13, 20]. Students often come into the capstone course feeling confident in their knowledge of functions. At the beginning of the course, the instructor administers an exam testing students' knowledge of the definition of a function, the difference between a function and a 1–1 function, and other function related questions found in the high school curricula.

Upon taking the exam, students leave a bit less confident in their knowledge of functions. Here are some of the misconceptions that students have had.

- A function is just an equation.
- A function is only a function if is has a continuous graph.
- A function has to be 1–1.
- All functions can be graphed.

The instructor endeavors to help students examine their own definition (or misconception) of function and then restructure it accordingly. Students are asked to write down their ''best'' definition of the concept of function. As noted above, students do not have a uniform view of the concept of function. The instructor records each definition offered on the chalkboard so that all of the students' definitions are available for discussion. Students are asked to identify the most important aspects of their definitions. The instructor then places a definition from the course textbook and asks students to note the similarities and differences between their definitions of function and the textbook's definition of function. Once preservice teachers have identified the similarities and differences between definitions, they have the opportunity to identify and discuss the ''big ideas'' associated with functions.

One big idea concerning functions is that of univalence [7, 8]. Univalence is the idea that given two sets, for there to be a function, every element in the first set must be assigned to a unique element in the second set via some rule. Another big idea is that functions can be arbitrary in the sense that they do not have to involve sets of numbers, or in other words, they can involve a correspondence between two sets of objects, such as pets and pet owners. Many preservice teachers are familiar with univalence, but are not familiar with the arbitrariness of functions. Some students have noted in end of course questionnaires that they had never had the opportunity to determine the essential qualities of a function. Taking time to identify and understand the two big ideas of functions helps the preservice teachers clarify their own misconceptions about functions and therefore better understand and explain the concept of functions to others [24].

4. STARTING TO THINK LIKE A TEACHER

Preservice teachers in the capstone course have had an entire academic career to learn mathematics, yet too often this has only meant remembering the material long enough to pass an examination. Knowing mathematics in a way that allows effective teaching it takes a different type of knowledge. Hill, Rowan, and Ball [11] define this type of knowledge as *mathematical knowledge for teaching*. They state,

> By ''mathematical knowledge for teaching,'' we mean the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this ''work of teaching'' include explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs. [11, p.373]

During the capstone course, the instructor endeavors to give preservice teachers opportunities to strengthen their mathematical knowledge for teaching.

One central activity used to help preservice teachers strengthen this knowledge is giving them the opportunity to teach part of the capstone course. Small groups (two to three) preservice teachers are assigned one section from the textbook to teach to their peers. It is done with the understanding that the level of teaching should be at the college level. Two weeks before the group teaches, they visit with the instructor to discuss the mathematics in their assigned section. It is the group's responsibility to review the section in the text and identify the ''big ideas'' prior to their visit with the professor. During the visit, the instructor helps the preservice teacher group with any misunderstandings that they might have. Former students in the capstone course have noted that they learn the mathematics more thoroughly in the section they are responsible for than any other section studied during the course.

One week before the preservice teacher group teaches the capstone course, they meet again with the instructor. This time the purpose of the meeting is to discuss the actual plan for the lesson. The group outlines the activities that they plan to use for the instructor. As part of the ensuing discussion they are asked to mathematically justify their choice of activities. The goal of the meeting is to have the group reflect on whether the activities they plan to use in their teaching will help students understand the big ideas of the section. In other words, are the activities mathematically appropriate for the objective of the lesson? Both the instructor and the preservice teacher group must be satisfied with the activities before the group can teach the lesson.

On the day that the preservice teacher group teaches, the instructor sits in the audience and takes notes. He observes the mathematics being taught and how the students in the class are coming to understand it. He also observes how the group's mathematical knowledge is brought out through the way the lesson is taught. Since this is most likely the first time the preservice teacher has ever taught, the atmosphere in the classroom is supportive. Sometimes, the instructor steps in to assist the preservice teacher if he or she is struggling mathematically. Also, sometimes the instructor asks questions that a high school student might ask in order to get the preservice teacher thinking and making decisions during the process of teaching.

The grading of the preservice teacher group's teaching is done in two stages. First, students are graded on how prepared they are when they meet with the instructor. Do students identify and understand the big ideas of the sections they will teach? Do students thoughtfully choose activities that will help their peers understand the big ideas? Next, students are graded on their presentation. Was it well organized? Did everyone from the preservice teacher group participate in leading the class? How did the group handle questions from the class? Finally, students receive a grade on their presentation of the big ideas. Did the preservice teacher group present the big ideas correctly? Is there evidence that the students understand the big ideas from the lesson? After the lesson is finished, the instructor gives his evaluation notes to the preservice teacher group and clarifies any doubts they might have about how the lesson was received.

5. STUDENT REACTIONS TO PARTICIPATION IN THE CAPSTONE COURSE

Even though the capstone course is new, it has already received positive reviews from students. The College of Science at UTEP formed a focus group of preservice high school mathematics teachers in order to discover which course, or courses, taught at UTEP had a significant impact on them as future teachers. The capstone course was the only mathematics course identified as having an impact on them as future teachers. In end-of-course interviews conducted by the instructor, students have reported positive effects from participating in the capstone course. One student said that participating in the capstone course helped her see that there were multiple solutions and representations in mathematics and that she needed to broaden her knowledge so that she would be able to understand students' reasoning. Another student said that he felt that when he started the course, he had gaps in his mathematical knowledge. He said that the capstone course filled in those gaps in his understanding. Another preservice teacher said that participating in the capstone course made her start to think about how high school students think and what examples would help them learn mathematics deeply.

The capstone course is far from perfect and continues to evolve and, hopefully, get better. The author still does not know the long-term effects of participation in the capstone course and one capstone course cannot make up for all the deficiencies that a preservice student might have before they teach. Yet, it seems clear that preservice high school teachers need more opportunities to think deeply about the mathematics they will teach, make connections between college and high school mathematics, and start to think like a teacher beginning the very first day of their college career. Providing a capstone course of the type described in this article for all preservice mathematics teachers is a step in this direction, a very important step in my experience.

REFERENCES

1. Borasi, R. 1994. Capitalizing on errors as ''springboards for inquiry'': A teaching experiment. *Journal for Research in Mathematics Education.* 25(2): 166–208.
2. Brown, C. A., and H. Borko. 1992. Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209–239). New York: Macmillan.

3. Carlsen, W. S. 1993. Teacher knowledge and discourse control: Quantitative evidence from novice biology teachers' classrooms. *Journal of Research in Science Teaching.* 30(5): 471–481.
4. Conference Board of the Mathematical Sciences (CBMS) 2001. *The mathematical education of teachers*. American Mathematical Society in cooperation with the Mathematics Association of America. Providence, RI: American Mathematical Society.
5. Cooney, T. J. 1994. Teacher education as an exercise in adaptation. In D. B. Aichele and A. F. Coxford (eds.), *Professional development for teachers of mathematics, 1994 Yearbook* (pp. 9–22). Reston, VA: National Council of Teachers of Mathematics.
6. Dubinsky, E. 1993. Computers in teaching and learning discrete mathematics and abstract algebra. In D. L. Ferguson (Ed.), *Advanced educational technologies for mathematics and science* (pp. 525–583). New York: Springer-Verlag.
7. Even, R. 1990. Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics.* 21: 521–544.
8. Even, R. 1993. Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education.* 24(2): 94–116.
9. Even, R., and D. Tirosh. 1995. Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject matter. *Educational Studies in Mathematics.* 29(1): 1–20.
10. Grouws, D. A., and K. A. Schultz. 1996. Mathematics teacher education. In J. P. Sikula, T. J. Buttery, and E. Gayton (eds.), *Handbook of research on teacher education: A project of the association of teacher educators* (2nd ed., pp. 442–458). New York: Simon and Shuster Macmillan.
11. Hill, H. C., B. Rowan, and D. L. Ball. 2005. Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal.* 42(2): 371–406.
12. Lacampagne, C. B., W. Blair, and J. Kaput. 1995. *The algebra initiative colloquium* (vol. 1). Washington, D.C.: U.S. Department of Education and Office of Educational Research and Improvement. (ERIC Document Reproduction Service No. ED 385 436).
13. Leinhardt, G., O. Zaslavsky, and M. K. Stein 1990. Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research.* 60: 1–64.
14. Leitzel, J. R. C. (Ed.). 1991. *A call for change: Recommendations for the mathematical preparation of teachers of mathematics*. Mathematical Association of America. Washington, DC: Mathematical Association of America.
15. Ma, L. 1999. *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

16. Mosenthal, J. H., and D. L. Ball. 1992. Constructing new forms of teaching: Subject matter knowledge in inservice teacher education. *Journal of Teacher Education.* 43(5): 347–356.
17. National Council of Teachers of Mathematics 1991. *Professional standards for teaching mathematics*. Reston, VA: NCTM.
18. National Council of Teachers of Mathematics 2000. *Principles and standards for school mathematics*. Reston, VA: NCTM.
19. Stein, M. K., J. A. Baxter, and G. Leinhardt. 1990. Subject-Matter knowledge and elementary instruction: A case from functions and graphing. *American Educational Research Journal.* 27(4): 639–663.
20. Tall, D. 1992. The transition to advanced mathematical thinking: Functions, limits, infinity, and proof. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 495–511). New York: Macmillan.
21. Usiskin, Z., A. Peressini, E. A. Marchisotto, and D. Stanley. 2003. *Mathematics for high school teachers: An advanced perspective*. Upper Saddle River, NJ: Prentice-Hall.
22. Van Dooren, W., L. Verschaffel, and P. Onghena. 2002. The impact of preservice teachers' content knowledge on their evaluation of students' strategies for solving arithmetic and algebra word problems. *Journal for Research in Mathematics Education.* 33(5): 319–351.
23. Wilson, M. R. 1994. One preservice secondary teacher's understanding of function: The impact of a course integrating mathematical content and pedagogy. *Journal for Research in Mathematics Education.* 25(4): 346–370.
24. Winsor, M. S. 2003. *Preservice mathematics teachers' knowledge of functions and its effect on lesson planning at the secondary level*. Unpublished Ph.D. dissertation. Iowa City: University of Iowa.
25. http://www.tea.state.tx.us/rules/tac/chapter111/ch111c.html). Last accessed 3/13/08.

BIOGRAPHICAL SKETCH

Matthew S. Winsor is an assistant professor at Illinois State University. He is interested in teachers' mathematical knowledge and how it affects their ability to teach mathematics in a conceptual manner. He is also interested in finding effective ways to help ELL students learn mathematics.