Let  $\mathcal{K}$  denote the set of  $2 \times 2$  matrices of the form  $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ , where  $a, b \in \mathbb{R}$ .

- 1. Show: If A and B are elements in K, then  $A + B \in K$ .
- 2. Show: If A and B are elements in K, then  $A \cdot B \in K$ .
- 3. Show that  $A \cdot B = B \cdot A$  holds for all elements  $A, B \in \mathcal{K}$ .
- 4. Note that the identity matrix  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  is an element of  $\mathcal{K}$ . Show: If the matrix  $A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$  in  $\mathcal{K}$  satisfies  $a^2 + b^2 \neq 0$ , then A has a matrix inverse. What is  $A^{-1}$ ? Does  $A^{-1}$  always lie in  $\mathcal{K}$ ?
- 5. Show that we can identify K with  $\mathbb{C}$ , i.e., find a bijection  $f: K \to \mathbb{C}$  such that f(A+B) = f(A) + f(B) and  $f(A \cdot B) = f(A) \cdot f(B)$ . (This is easier than it sounds.)
- 6. Under this identification, find the element I in K that corresponds to  $i \in \mathbb{C}$  (not surprisingly, there are actually two possible choices). Compute  $I \cdot I$ .
- 7. Under this identification, what is the "conjugate" of an element in  $\mathcal{K}$ , what is the "modulus" of an element in  $\mathcal{K}$ ?